Lesson 20 – Differential Equations – Separation of Variables Part II

I. Warm Up

II. Word Problems

Announcements

Test 2 – Monday

See announcement in Brightspace

Arrive by 6:15 PM

① Come to front, pick up Scantron

② Go to seat, fill out scantron
+ put photo ID on desk
+ calculator

I Warm-up

$\boxed{Ex}$ $x^2 y' = 9y' + \frac{2x}{y^4}$

$$x^2 \frac{dy}{dx} = 9\frac{dy}{dx} + \frac{2x}{y^4}$$

$$x^2 \frac{dy}{dx} - 9\frac{dy}{dx} = \frac{2x}{y^4}$$

$$\frac{dy}{dx}(x^2 - 9) = \frac{2x}{y^4}$$

$$\frac{dy}{dx} = \frac{2x}{y^4(x^2-9)}$$

$$\frac{dy}{dx} = \frac{1}{y^4} \cdot \frac{2x}{x^2-9}$$

$$\int y^4\,dy = \int \frac{2x}{x^2-9}\,dx$$

$u = x^2 - 9$

$du = 2x\,dx$

$$\frac{y^5}{5} = \int \frac{du}{u}$$

$$\frac{y^5}{5} = \ln(|u|) + C$$

$$\frac{y^5}{5} = \ln(|x^2-9|) + C$$

$$y^5 = 5\ln(|x^2-9|) + \overset{(5C)}{C}$$

$$y = \sqrt[5]{5\ln(|x^2-9|)+C}$$

II. Word Problems

Recall: From Lesson 19

① A quantity y grows at a rate proportional to z if

$$\frac{dy}{dt} = kz$$

for some constant k.

A quantity y grows at a rate inversely proportional to z if

$$\frac{dy}{dt} = \frac{k}{z}$$

for some constant k.

$\boxed{\text{EX}}$ In a chemical reaction, chemical A is converted into chemical B at a rate that is proportional to the square of the amount of chemical A remaining in the system @ time t. If the system initially contains 10 g of chemical A and 1 hour later contains 6 g of chemical A, how much of chemical A remains after 5 hours?

a = amount of chemical A in system after t hours $a(t)$

$$\left.\begin{aligned} a(0) &= 10 \\ a(1) &= 6 \end{aligned}\right\} \text{initial values}$$

$$a(5) = ?$$

$$\frac{da}{dt} = ka^2$$

$$\int \frac{da}{a^2} = \int k\,dt$$

$$\int a^{-2}\,da = \int k\,dt$$

$$\frac{a^{-1}}{-1} = kt + C$$

$$\frac{-1}{a} = kt + C$$

$a(0) = 10$

$$\frac{-1}{10} = k(0) + C$$

$$C = \frac{-1}{10}$$

$$\frac{-1}{a} = kt - \frac{1}{10}$$

$a(1) = 6$

$$\frac{-1}{6} = k(1) - \frac{1}{10}$$

$$k = \frac{-1}{6} + \frac{1}{10} = \frac{-5}{30} + \frac{3}{30} = \frac{-2}{30}$$

$$= \frac{-1}{15}$$

$$\frac{-1}{a} = \frac{-1}{15}t - \frac{1}{10}$$

$$\frac{-1}{a} = -\frac{1}{15}t - \frac{1}{10}$$

$$\frac{1}{a} = \frac{1}{15}t + \frac{1}{10}$$

$$a = \frac{1}{\frac{1}{15}t + \frac{1}{10}} \cdot \frac{30}{30}$$

$$\overset{= a(t)}{a} = \frac{30}{2t+3}$$

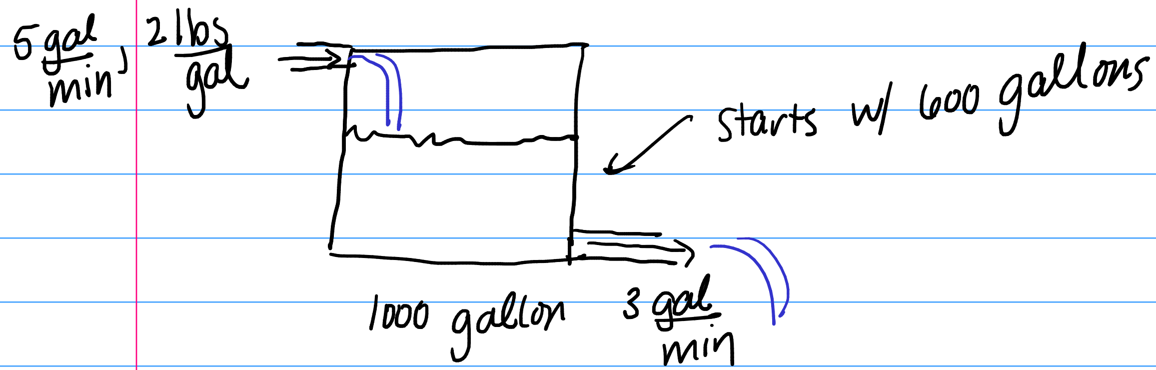
$$a(5) = \frac{30}{2(5)+3} = \frac{30}{13} \approx 2.3077\,g$$

$\boxed{Ex}$ (Mixture Problem)

A 1000-gallon tank initially contains 600 gallons of brine (salt water solution) that contains 150 pounds of disolved salt.

Brine containing 2 lbs of salt per gallon flows into the tank at a rate of 5 gallons per minute. The well-stirred mixture flows out of the tank at a rate of 3 gallons per minute.

Set up a differential equation for the amount of salt A(t) in the tank @ time t.



A(t) = amt of salt in tank @ time t.

units $\frac{lbs}{min}$

$$\frac{dA}{dt} = \text{rate in of salt} - \text{rate out of salt}$$

rate in = $\frac{5 \cancel{\text{ gal of brine}}}{\text{min}} \times \frac{2 \text{ lbs salt}}{\cancel{\text{gal of brine}}} = 10 \frac{\text{lbs salt}}{\text{min}}$

rate out: A(t) = amt of salt in tank @ time t
V(t) = volume of mixed brine @ time t

$$\text{rate out} = \frac{3\text{ gal}}{\text{min}} \times \frac{A(t)\text{ lbs of salt in tank}}{V(t)\text{ gallons of brine in tank}}$$

$$= 3 \times \frac{A}{600 + \underbrace{5t}_{\text{amt of brine that came in}} - \underbrace{3t}_{\text{amt of brine that flowed out}}}$$

$$\text{rate out} = \frac{3A}{600+2t}$$

D.E. $$\frac{dA}{dt} = 10 - \frac{3A}{600+2t}$$

IVP $$\frac{dA}{dt} = 10 - \frac{3A}{600+2t} \qquad A(0) = 150$$